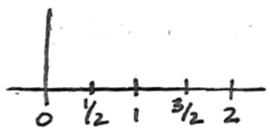
AP Calculus AB

Approx. Area Using Riemann Sums

1) $f(x) = x^2$ on $[0,2]$



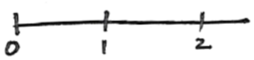
$\Delta x = 1/2$

a) $L_4 = f(0)\cdot\frac{1}{2} + f(1/2)\cdot\frac{1}{2} + f(1)\cdot 1/2 + f(3/2)\cdot\frac{1}{2}$
$= \frac{1}{2}\left[0 + \frac{1}{4} + 1 + \frac{9}{4}\right]$

b) $R_4 = f(2)\cdot\frac{1}{2} + f(3/2)\cdot\frac{1}{2} + f(1)\cdot 1/2 + f(\frac{1}{2})\cdot\frac{1}{2}$
$= \frac{1}{2}\left[4 + \frac{9}{4} + 1 + \frac{1}{4}\right]$

c) $M_4 = f(\frac{1}{4})\cdot\frac{1}{2} + f(\frac{3}{4})\cdot\frac{1}{2} + f(\frac{5}{4})\cdot\frac{1}{2} + f(\frac{7}{4})\cdot\frac{1}{2}$
$= \frac{1}{2}\left[\frac{1}{16} + \frac{9}{16} + \frac{25}{16} + \frac{49}{16}\right]$

2) $f(x) = x^3$ on $[0,2]$

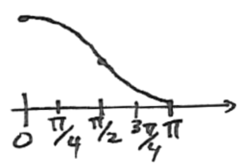


a) $L_4 = f(0)\cdot\frac{1}{2} + f(\frac{1}{2})\cdot\frac{1}{2} + f(1)\cdot\frac{1}{2} + f(\frac{3}{2})\cdot\frac{1}{2}$
$= \frac{1}{2}\left[0 + \frac{1}{8} + 1 + \frac{27}{8}\right]$

b) L_4 is an underestimate of the actual are because $f(x)$ is increasing on $[0,2]$.

3) $f(x) = 1 + \cos x$ on $[0,\pi]$

$\Delta x = \frac{\pi}{4}$

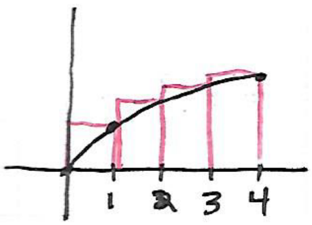


a) $R_4 = f(\pi)\cdot\frac{\pi}{4} + f(\frac{3\pi}{4})\cdot\frac{\pi}{4} + f(\frac{\pi}{2})\cdot\frac{\pi}{4} + f(\frac{\pi}{4})\cdot\frac{\pi}{4}$
$= \frac{\pi}{4}\left[0 + 1 - \frac{\sqrt{2}}{2} + 2 + 1 + \frac{\sqrt{2}}{2}\right]$

b) R_4 is an underestimate because $f(x)$ is decreasing

4) $f(x) = \sqrt{x}$ on $[0,4]$

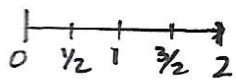
$\Delta x = \frac{4-0}{4} = 1$



a) $R_4 = f(4)\cdot 1 + f(3)\cdot 1 + f(2)\cdot 1 + f(1)\cdot 1$
$= 2 + \sqrt{3} + \sqrt{2} + 1$

b) R_4 is an overestimate because $f(x)$ is increasing.

5) $f(x) = (x-1)^2$ on $[0,2]$



$\Delta x = \frac{2}{4} = \frac{1}{2}$

$M_4 = f(\frac{1}{4})\cdot\frac{1}{2} + f(\frac{3}{4})\cdot\frac{1}{2} + f(\frac{5}{4})\cdot\frac{1}{2} + f(\frac{7}{4})\cdot\frac{1}{2}$
$= \frac{1}{2}\left[(\frac{1}{4}-1)^2 + (\frac{3}{4}-1)^2 + (\frac{5}{4}-1)^2 + (\frac{7}{4}-1)^2\right]$

6) $f(x) = (x^2-2x-1)^{2/3}$

$f'(x) = \frac{2}{3}(x^2-2x-1)^{-1/3}\cdot(2x-2)$

$f'(0) = \frac{2}{3}(-1)(-2) = \frac{4}{3}$